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TECHNICAL NOTE 4198

EFFECTS OF AIRPLANE FLEXIBILITY ON WING STRAINS IN
ROUGH AIR AT 35,000 FEET AS DETERMINED BY A
FLIGHT INVESTIGATION OF A LARGE
SWEPT-WING AIRPLANE

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SUMMARY

A flight investigation has been made on a large sweptback-wing bomber airplane in rough air at an altitude of 35,000 feet in order to determine the effects of wing flexibility on wing bending and shear strains and to compare the results with results previously obtained at low altitude (5,000 feet) and reported in NACA Technical Note 4107. The effects of wing flexibility on the wing strains were, on the average, about 20 percent larger at the higher altitude. Representative values of the amplification factors varied from about 1.3 at the root stations to about 2.5 at the midspan stations.

INTRODUCTION

Flight investigations of the effects of airplane flexibility on the wing strains that develop during flight through rough air have shown that substantial amplifications of the strains may occur. (See, for example, refs. 1 to 5.) Analytical methods for calculating the structural response of unswept-wing airplanes to atmospheric turbulence involving simple wing-bending modes have been developed and are reported in references 6 to 8, and results of these calculations show good correlation with the results of flight-test studies for the unswept-wing airplanes considered. For swept-wing airplanes, however, the responses in rough air are likely to be more complicated since the structural response of a swept-wing airplane may be expected to involve significant effects of torsion on the airplane aerodynamics, on the stability of the airplane, and on the structural strains. Flight tests were, therefore, undertaken in order to obtain information on the magnitude and character of the effects of flexibility and aeroelasticity on the strains in rough air for the case of a flexible sweptback-wing airplane and to provide experimental data for comparison with analytical results.

An analysis of the flight-test measurements made at an altitude of 5,000 feet and a Mach number of approximately 0.63 is presented in reference 5. The results of the analysis of reference 5 indicate that both dynamic and static aeroelastic effects have a large influence on the wing bending and shear strains. The bending-strain amplification factors reflecting the dynamic effects alone were found to vary from approximately 1.25 at the root to 2.7 at the 0.60-semispan station.

Inasmuch as the effects of flexibility might be expected to increase with altitude because of the decreased aerodynamic damping, the flight tests of the present investigation were made at an altitude of 35,000 feet and a Mach number of 0.64. An analysis of the high-altitude test data is presented, and the strain amplification factors obtained are compared with those given in reference 5 for the tests at an altitude of 5,000 feet.

SYMBOLS

g	acceleration due to gravity, 32.2 ft/sec ²
a_n	normal acceleration, g units unless otherwise noted
q	dynamic pressure, $\frac{\rho V^2}{2}$, lb/sq ft
b	airplane wing span, ft
y	distance along span measured perpendicular to airplane center line, ft
σ_F	root-mean-square deviation for flexible airplane
σ_R	root-mean-square deviation for rigid airplane
ρ	density of air, slugs/cu ft
V	true airspeed, ft/sec
$\Phi_{a_n}(f)$	power-spectral-density function of normal acceleration, $\lim_{T \rightarrow \infty} \frac{1}{T} \left \int_{-T}^T a_n(t) e^{-i2\pi f t} dt \right ^2$
f	frequency, cps

T specified time, sec

t time, sec

AIRPLANE INSTRUMENTATION AND TESTS

The airplane and the instrumentation for the present tests are the same as those described in reference 5. For convenience, however, a brief description of the instrumentation pertinent to the present investigation follows.

(1) An NACA air-damped recording accelerometer (response essentially flat to about 10 cps, accuracy $\pm 0.0125g$) was mounted within 2 feet of the center of gravity of the airplane to measure normal acceleration.

(2) Electrical wire-resistance strain gages connected as four active gages in a bridge circuit were installed on the wing spars at the eight locations on the semispan shown in figure 1. The gages were not calibrated to measure actual load but served to give only local strain indications.

(3) An NACA airspeed-altitude recorder provided a record of airspeed and pressure altitude.

(4) NACA control-position recorders were used to obtain the aileron, rudder, and elevator displacements during the gust runs. These records were used to monitor the control movements in order to insure that the airplane response in rough air was not influenced by the pilot.

The film speed of the acceleration and airspeed-altitude recorders was approximately $1/4$ inch per second, and the film speed of the oscillographs that were used to record the strain-gage outputs was approximately 1 inch per second. All recordings were correlated by means of an NACA $\frac{1}{10}$ -second chronometric timer.

Approximately 90 seconds of strain and acceleration time-history data taken during flight in light, clear-air turbulence were available for analysis. The average Mach number for the tests was 0.64, and the pressure altitude was 35,000 feet. The average aircraft weight was 112,000 pounds, and the center of gravity was located at 20 percent of the mean aerodynamic chord. (This condition is a low-weight condition for this airplane and is approximately the same as for the test of ref. 5.)

GENERAL METHOD OF ANALYSIS

The method used for determining the effects of flexibility on the wing strains in rough air was essentially the same as that used in reference 5. The method basically involves comparisons of the strains per unit load in rough air with the strains per unit load in slow pull-up maneuvers. In order to separate the purely dynamic or vibratory effects of airplane flexibility on the strains from the combined dynamic and static aeroelastic effects, the following two procedures are employed. First, the strains measured in rough air are compared with the strains measured for the same total aerodynamic loadings applied statically in slow pull-up maneuvers performed in smooth air at the same dynamic pressure and weight condition as the tests in rough air. Since the effects of static aeroelasticity are reflected to somewhat the same extent in the strains measured in both the rough-air and smooth-air tests, this comparison provides a measure of the purely dynamic or vibratory effects of airplane flexibility on the strains. Second, the strains in rough air are compared with the strains resulting from the static application of the same loads to a "rigid" airplane; that is, an airplane embodying no static aeroelastic effects. The reference strains for the "rigid" airplane are obtained by extrapolating the values of strain per unit load measured in slow pull-ups to zero dynamic pressure, where the static aeroelastic effects are minimized. The ratio of the strain in rough air to this reference strain provides a measure of the combined dynamic and static aeroelastic effects on the strains.

For both procedures, the acceleration measured at the center-of-gravity location is used directly as a measure of the loading on the airplane in the pull-up maneuvers and with some modification (as is discussed later) is also used in conjunction with the tests made in rough air. As in reference 5, two measures of the magnitude of the flexibility effects termed "amplification factors" are employed; one is based on comparisons of counts of peak strains, and the other is based on comparisons of root-mean-square strains.

EVALUATION OF DATA AND RESULTS

The data-reduction procedures for the various time histories of strain and acceleration involved the following steps: (1) an evaluation of the wing strains experienced in rough air; (2) an evaluation of the reference acceleration in rough air; (3) the use of the steady strains per unit acceleration obtained in pull-up maneuvers. The procedures used for each of these steps and the results obtained are described in

order. The recorded quantities were read at 0.05-second intervals along the time histories, and the incremental values (that is, fluctuations from the steady level-flight value) were determined for each time history. These 0.05-second readings were then considered to be an adequate representation of the time histories and were used in the remainder of the data evaluation.

Wing Strains in Rough Air

In order to compare the overall strain and acceleration time histories in terms of the number of peaks of a given magnitude, the strain peaks were first counted from the time histories, grouped into class intervals, and then formed into cumulative frequency distributions. Figure 2 is an illustrative time history showing the method of making the peak counts. The peaks which were counted are indicated by the letters a, b, c, and d. As can be seen from the sketch, only one peak was counted between consecutive intersections of the trace with the trace position for steady level flight. Only the peaks exceeding a given threshold level, as indicated in figure 2, were evaluated. (The threshold level depended upon the sensitivity of the individual gage.) In addition to the determination of the cumulative peak distributions, the time histories were used to obtain the root-mean-square strains.

In order to compare the strains obtained in rough air with the strains obtained for the same loadings applied in slow pull-up maneuvers and to facilitate comparisons between the strains at different stations the strains were converted, for convenience, to equivalent acceleration units. This conversion was accomplished by dividing all strain indications by the strain indication per g in steady pull-ups (hereafter referred to as pull-up factor) for the individual gages. Two sets of pull-up factors corresponding to the rough-air test condition ($q = 145 \text{ lb/sq ft}$) and to the $q = 0 \text{ lb/sq ft}$ reference condition were used. The cumulative frequency distributions of strain peaks in acceleration units for the $q = 145 \text{ lb/sq ft}$ reference condition only are shown by the solid curves of figures 3 and 4 for both the bending and shear strains, respectively.

Reference Acceleration in Rough Air

For the low-altitude investigation of reference 5, the center-of-gravity acceleration was used as the reference acceleration since the first wing bending mode appeared to have little effect and the effects of higher modes could easily be faired out. In order to determine the adequacy of the center-of-gravity acceleration for use as the reference acceleration for the present tests, a power spectrum of the faired

center-of-gravity acceleration was obtained and is given in figure 5 together with the spectrum for the low-altitude tests. From the relative areas of the humps in the spectra at a frequency of approximately 1.3 cycles per second, which is approximately the frequency of the fundamental wing bending mode, it appears that the fundamental wing vibratory mode affects the center-of-gravity acceleration considerably more at the higher altitude. In order to use the center-of-gravity acceleration as a reference for the present data, therefore, the effect of this first mode on both the root-mean-square acceleration and the peak accelerations has to be removed.

A simple correction was made to the root-mean-square value of center-of-gravity acceleration based on the assumption that the area under the hump (shown by the hatched area in fig. 5), as compared with the total area under the spectrum, represents the relative contribution of the first wing bending mode to the total mean square. This adjustment reduced the root-mean-square value by approximately 7.5 percent.

In addition to the root-mean-square value of acceleration, peak counts of the faired center-of-gravity acceleration time history were obtained for purposes of comparison with the peak strains. The peak counts were made for the 90-second rough-air test in a manner similar to the counts of rough-air strain, as illustrated in figure 2. These peak readings were then used to determine a cumulative frequency distribution as was done for the strain time histories. In order to estimate the magnitude of the effect of the fundamental wing bending mode on the cumulative frequency distribution, the relation between the distribution of peak values and the spectrum of a stationary Gaussian random disturbance was used. This relation is given in reference 9 as

$$N(y) = \frac{1}{2\pi} \left[\frac{\int_0^\infty \omega^2 \Phi(\omega) d\omega}{\int_0^\infty \Phi(\omega) d\omega} \right]^{1/2} e^{-y^2/2\sigma^2}$$

$$= N_0 e^{-y^2/2\sigma^2} \quad (1)$$

where

$N(y)$ average number of maximums per second exceeding given values of y

ω frequency, radians/sec

$\Phi(\omega)$ power-spectral-density function of random disturbance $y(t)$

$$\sigma^2 = \int_0^{\infty} \Phi(\omega) d\omega$$

$$N_0 = \frac{1}{2\pi} \left[\frac{\int_0^{\infty} \omega^2 \Phi(\omega) d\omega}{\int_0^{\infty} \Phi(\omega) d\omega} \right]^{1/2}$$

For the type of peak count used herein, equation (1) yields a good approximation for the number of maximums above a given value of y .

In order to apply equation (1) to the present analysis, let the subscript 1 designate the various quantities associated with the measured cumulative frequency distribution and spectrum of acceleration, and let the subscript 2 designate the various quantities of equation (1) obtained for the modified spectrum (that is, with the first-mode effects faired out of the spectrum as shown in fig. 5). Then equation (1) may be used to show that for any value of y_1 the value of y_2 , which is exceeded with equal frequency $N(y)$, is given by

$$y_2 = \left[\frac{\sigma_2^2}{\sigma_1^2} + \frac{2\sigma_2^2}{y_1^2} (\log_e N_{0,2} - \log_e N_{0,1}) \right]^{1/2} y_1 \quad (2)$$

Equation (2) thus permits the adjustment of the measured cumulative frequency distribution of acceleration for the distortion effects introduced by the presence of the first mode on the center-of-gravity accelerations. The measured distribution was modified in this manner, and the "reference acceleration" distribution obtained is given in figures 3 and 4. In order to indicate the magnitude of this effect, the measured cumulative distribution, together with the modified distribution, is presented in figure 6. The modification reduced the acceleration values for a given cumulative frequency by about 10 percent at the higher levels of acceleration and by an increasingly larger percentage with decreasing acceleration level.

Strain Per. g From Pull-Ups in Smooth Air

In order to determine a reference strain indication per g (or pull-up factor) for the various gages for a "statically" applied load, as discussed in the "General Method of Analysis," use was made of data obtained in the investigation of reference 5 for the pull-up maneuvers. These data of reference 5 are usable for the present tests, since the range of dynamic pressure covered includes the dynamic pressure of the present high-altitude rough-air tests, and the average airplane weight during the tests was about the same as for reference 5. A minor adjustment which amounted to less than 2 percent was made, however, because of the differences in weight between the two tests. A typical plot of the strain indication per g against dynamic pressure is presented in figure 7 for wing station 414. The value of dynamic pressure for the rough-air test also is indicated in the figure. As shown by solid lines in the figure, the variation of strain indication per g with dynamic pressure has been extrapolated to zero dynamic pressure. At zero dynamic pressure, a value of pull-up factor is obtained which is assumed to correspond to that which would be obtained if no load alleviation due to quasi-static wing twist has occurred. Two pertinent values of pull-up factor are thus obtained for each gage, one for zero dynamic pressure (a condition where quasi-static twist effects are eliminated) and the other for the dynamic pressure of the rough-air test (145 lb/sq ft). These two sets of values of pull-up factors are given in table I and were used to obtain amplification factors.

Amplification Factors

Two methods were used to determine amplification factors based on the pull-up factors given in table I. The pull-up factors for the two reference values of dynamic pressure just discussed were used for each of these two methods. In the first method, the amplification factor was determined from an overall comparison of the strain and reference acceleration histories in terms of the number of peaks of a given magnitude. This comparison was made by use of the cumulative-frequency plots, such as figures 3 and 4, at a cumulative-frequency level corresponding to two times the root-mean-square strain in rough air for the various strain gages. (For example, for reference $q = 145$ lb/sq ft the amplification factor for the front-spar bending strain, figure 3(c), is obtained by dividing the abscissa value at point A by the value at point B.) In the second method, the amplification factor is defined as the ratio of the root-mean-square strain (in equivalent g units) to the root-mean-square reference acceleration.

Amplification factors determined for the bending and shear strains are given in table I and are shown in figures 8 and 9 as a function of

wing station for the bending and shear strains, respectively. For the front-spar shear gages at stations 54 and 252, reliable values of strain per g in pull-ups were not obtained; consequently, amplification factors for these two locations are not shown.

In order to compare the present results with the results of the tests at low altitude which were presented in reference 5, amplification factors based on the ratios of root-mean-square values from reference 5 are presented in figures 10 and 11 for the bending and shear strains, respectively, together with the results of the present tests.

DISCUSSION

Examination of the amplification factors given in figure 8 for the front- and rear-spar bending strain shows that, in general, the values are smallest at the wing root station, increase to a maximum at the 0.60-semispan station, and then decrease somewhat. When the amplification factors for the front spar are considered first, figure 8 and table I indicate that amplification factors obtained from a ratio of the root mean squares and based on the test dynamic-pressure pull-up factors ($q = 145$ lb/sq ft) increase from a value of 1.32 at the root to a value of 2.21 at station 414 and then decrease somewhat at the most outboard station. The amplification factors based on the strain values at $2\sigma_F$ show a similar variation along the span but have consistently higher values than those based on root-mean-square strain values. The same general variations along the span exist for the rear spar.

Inspection of figure 8 shows that the amplification factors which are based on the $q = 0$ reference condition and which provide a measure of the combined effects of dynamic amplification and static aeroelasticity on the strains vary along the span in a manner similar to the dynamic amplification factors based on the test dynamic-pressure reference condition but are somewhat smaller. This reduction in amplification factor is a reflection of the strain alleviation associated with the static aeroelastic effects.

Comparisons of the dynamic amplification factors (circled points) obtained for the tests at 35,000 feet and at 5,000 feet of altitude, based on the ratio σ_F/σ_R , show that the amplification factors at the high altitude have a variation along the span similar to the values at low altitude. (See fig. 10.) In addition, the amplification factors for the high-altitude tests are, on the average, about 20 percent higher than those for the low-altitude tests except for the most outboard station where the values of dynamic amplification factor for the two altitudes are essentially the same. The amplification factor at the root

station, for example, increased from about 1.1 at low altitude to about 1.3 at high altitude. The higher amplification factors were probably obtained because the fundamental wing bending mode was excited to a greater extent as a result of the decrease in aerodynamic damping at the higher altitude of the present tests.

As previously indicated, the reduction in amplification factors obtained by using the reference condition for the hypothetical "rigid" airplane ($q = 0$), as compared with the amplification factors obtained by using the test dynamic-pressure reference condition, is a reflection of the strain alleviation associated with the static aeroelastic effects. Consideration of the results of figure 10 indicates that the strain alleviation was considerably smaller for the present tests at an altitude of 35,000 feet than for the tests at an altitude of 5,000 feet. This result was to be expected, inasmuch as the amount of static alleviation decreased with decreasing q . This reduction in the static alleviation, when coupled with the larger dynamic strain amplification at the higher altitude, indicates that the overall effects of flexibility on strains are considerably worsened at high altitudes.

Inasmuch as the shear-strain results summarized in figures 9 and 11 follow the same general patterns as the results for the bending strains, no separate discussion is given.

CONCLUDING REMARKS

Data on wing bending and shear strains obtained from flight tests of a sweptback-wing airplane in rough air at an altitude of approximately 35,000 feet have been evaluated to supplement the flight-test data obtained in rough air at 5,000 feet of altitude and evaluated in NACA Technical Note 4107. The strain amplifications obtained at the high altitude varied along the span in a manner similar to the amplifications obtained at low altitude, with moderate amplifications at the wing root and very large strain amplifications over the midspan stations. From the overall viewpoint, the dynamic strain amplifications were roughly 20 percent higher for the present tests than those reported in NACA Technical Note 4107 for the tests at low altitude. Representative values of amplification factors varied from about 1.3 at the root stations to about 2.5 at the midspan stations. In addition, the relieving effects on the strains arising from static aeroelastic effects, which were large at the low altitude because of the high dynamic pressure, were considerably reduced at the high altitude because of the low dynamic pressure.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., October 23, 1957.

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TABLE I.- AMPLIFICATION FACTORS OF STRAIN

Wing station	Spar	Bending-strain indication				Shear-strain indication			
		Pull-up factor	$2\sigma_F$ level	Amplification factor		Pull-up factor	$2\sigma_F$ level	Amplification factor	
				Values at $2\sigma_F$ level	$\frac{\sigma_F}{\sigma_R}$			Values at $2\sigma_F$ level	$\frac{\sigma_F}{\sigma_R}$
(a) (b) (a) (b)									
Reference dynamic pressure, $q = 145 \text{ lb/sq ft}$									
54	Front	0.539	0.143	1.50	1.32	-----	-----	-----	-----
54	Rear	.966	.144	1.61	1.33	0.527	0.125	1.31	1.16
252	Front	.595	.168	1.86	1.55	-----	-----	-----	-----
252	Rear	.548	.173	1.95	1.60	.288	.176	2.34	1.63
414	Front	.578	.239	3.01	2.21	.424	.167	1.94	1.54
414	Rear	.600	.227	2.73	2.10	.198	.223	3.03	2.06
572	Front	.258	.176	2.51	1.63	.527	.143	1.46	1.32
572	Rear	.358	.146	1.94	1.35	.207	.183	3.05	1.69
Reference dynamic pressure, $q = 0 \text{ lb/sq ft}$									
54	Front	0.573	0.135	1.41	1.24	-----	-----	-----	-----
54	Rear	1.038	.134	1.50	1.24	0.544	0.121	1.27	1.12
252	Front	.664	.151	1.67	1.39	-----	-----	-----	-----
252	Rear	.601	.158	1.78	1.46	.336	.151	2.00	1.40
414	Front	.651	.212	2.67	1.96	.472	.150	1.74	1.38
414	Rear	.642	.212	2.55	1.96	.214	.206	2.80	1.91
572	Front	.290	.157	2.24	1.45	.573	.132	1.34	1.21
572	Rear	.406	.129	1.71	1.19	.229	.165	2.76	1.53

(a) Record deflection, inches per g (adjusted for changes in system voltage).

(b) Converted to equivalent g units by use of pull-up factor.

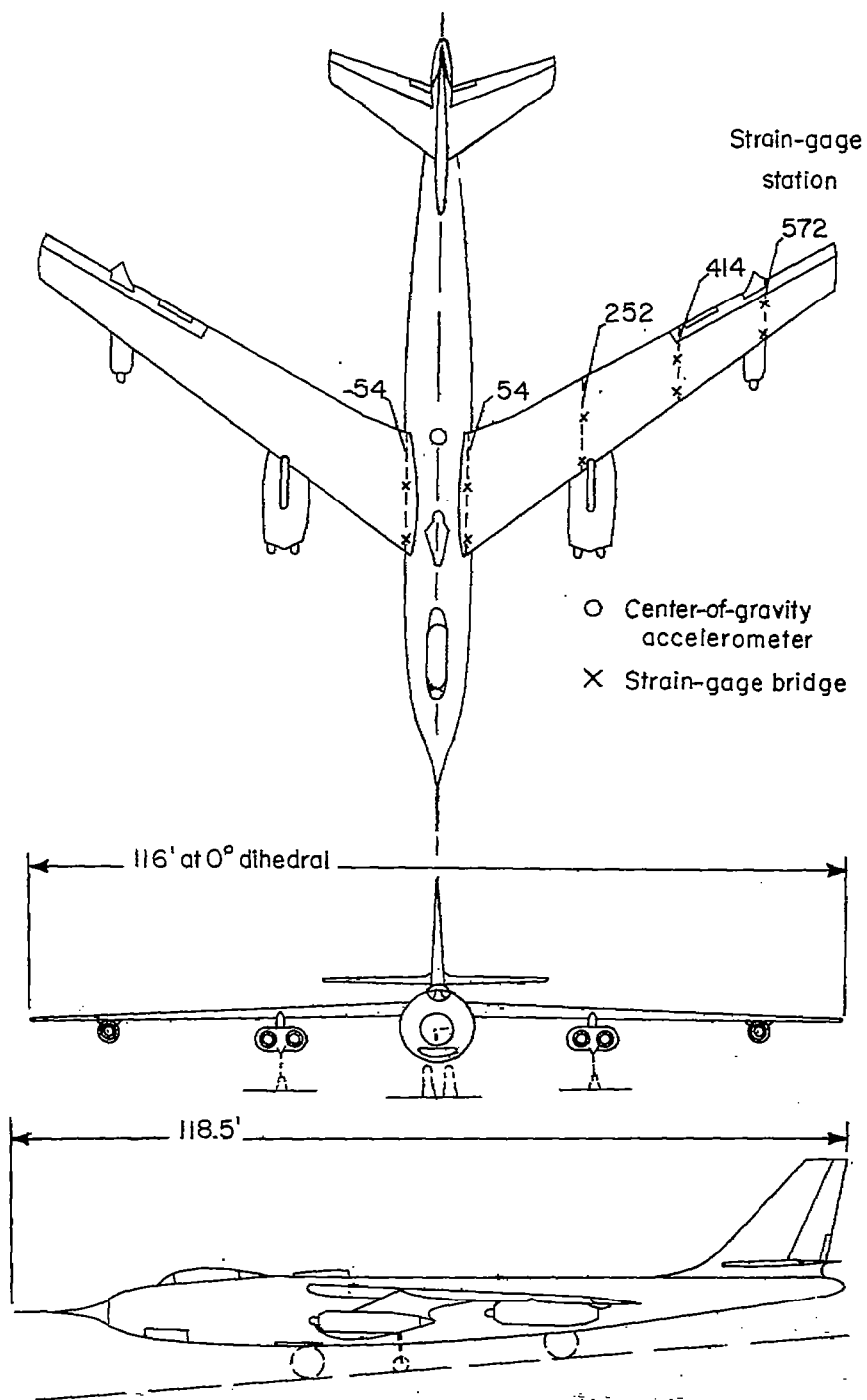


Figure 1.- Three-view drawing of test airplane.

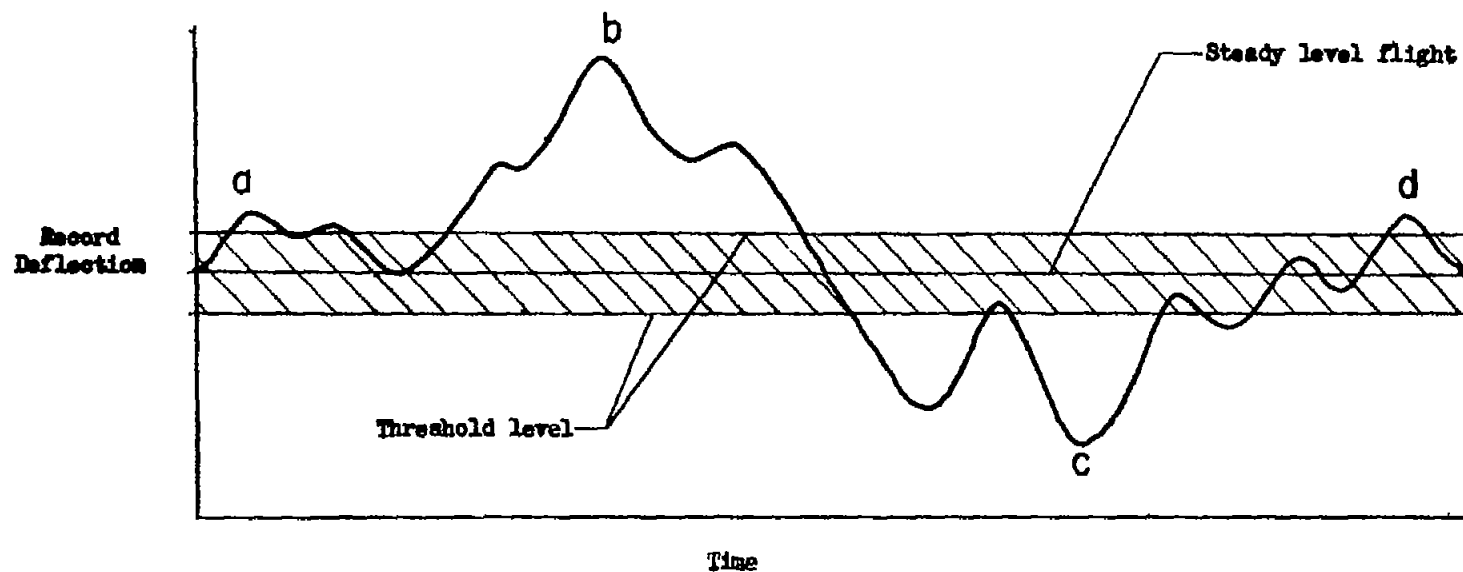


Figure 2.- Illustrative time history showing method of peak count. Peaks are represented by a, b, c, and d.

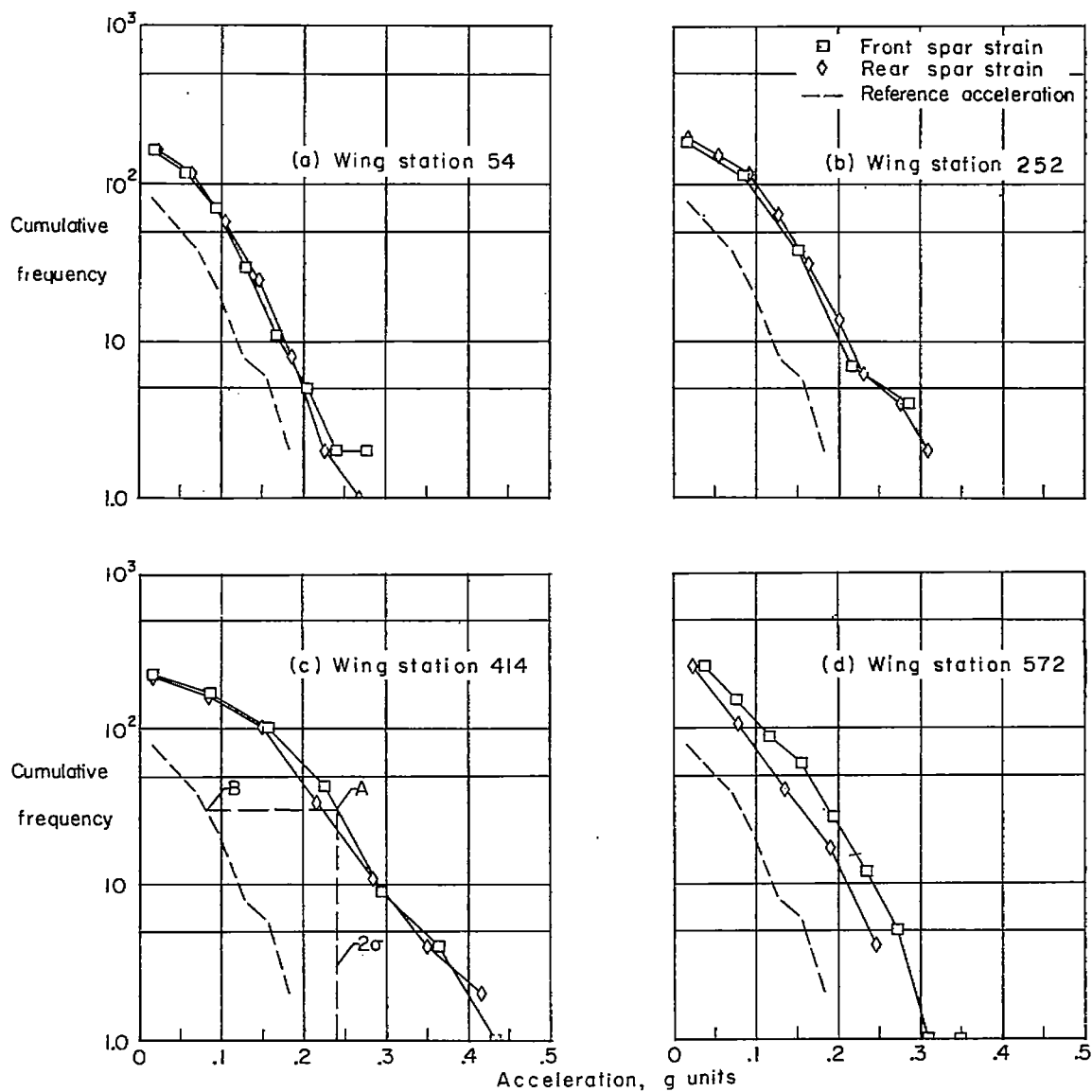


Figure 3.- Cumulative frequency distributions of reference acceleration and bending strains in g units. Reference $q = 145$ lb/sq ft.

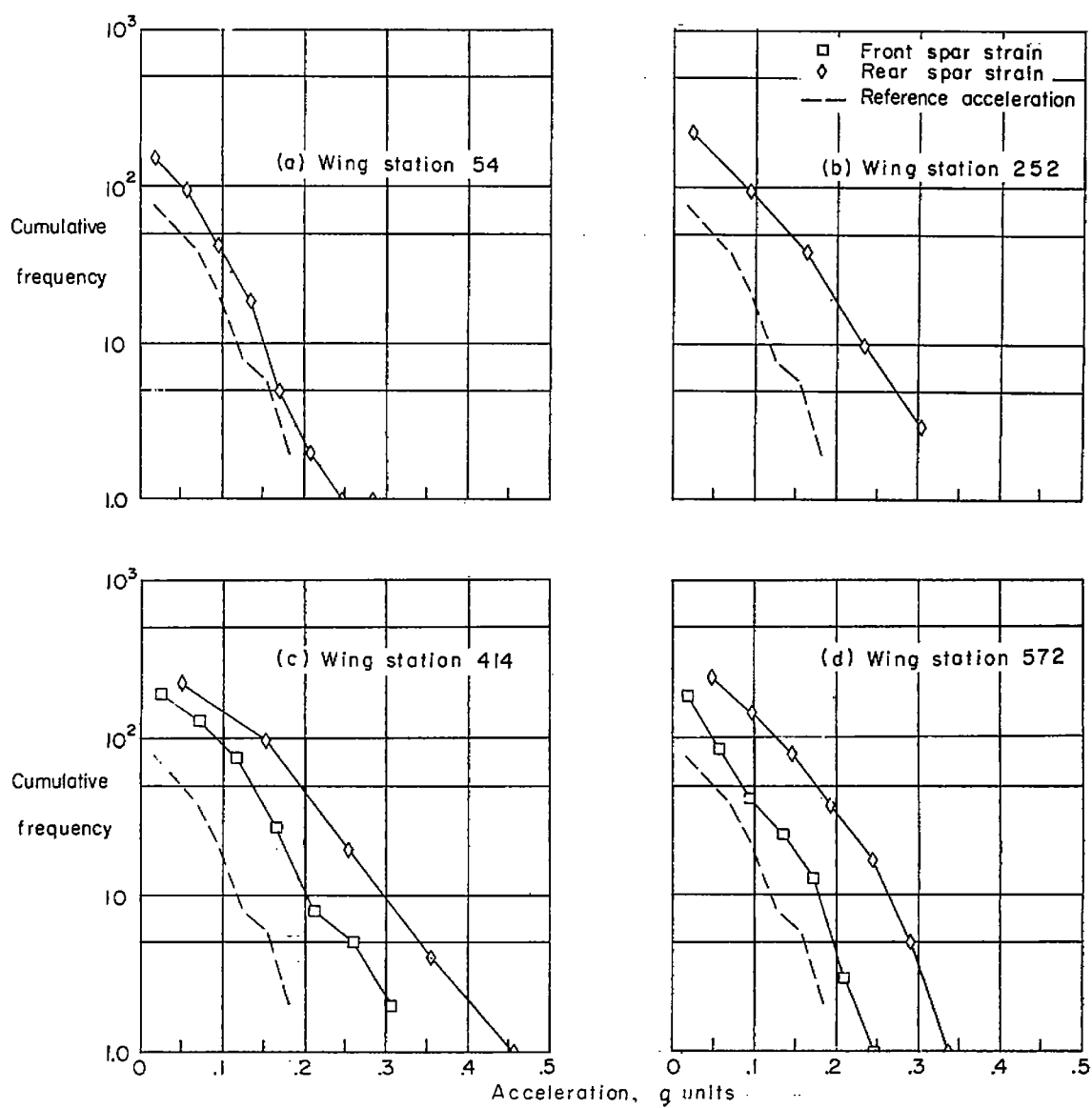


Figure 4.- Cumulative frequency distributions of reference acceleration and shear strains in g units. Reference $q = 145$ lb/sq ft.

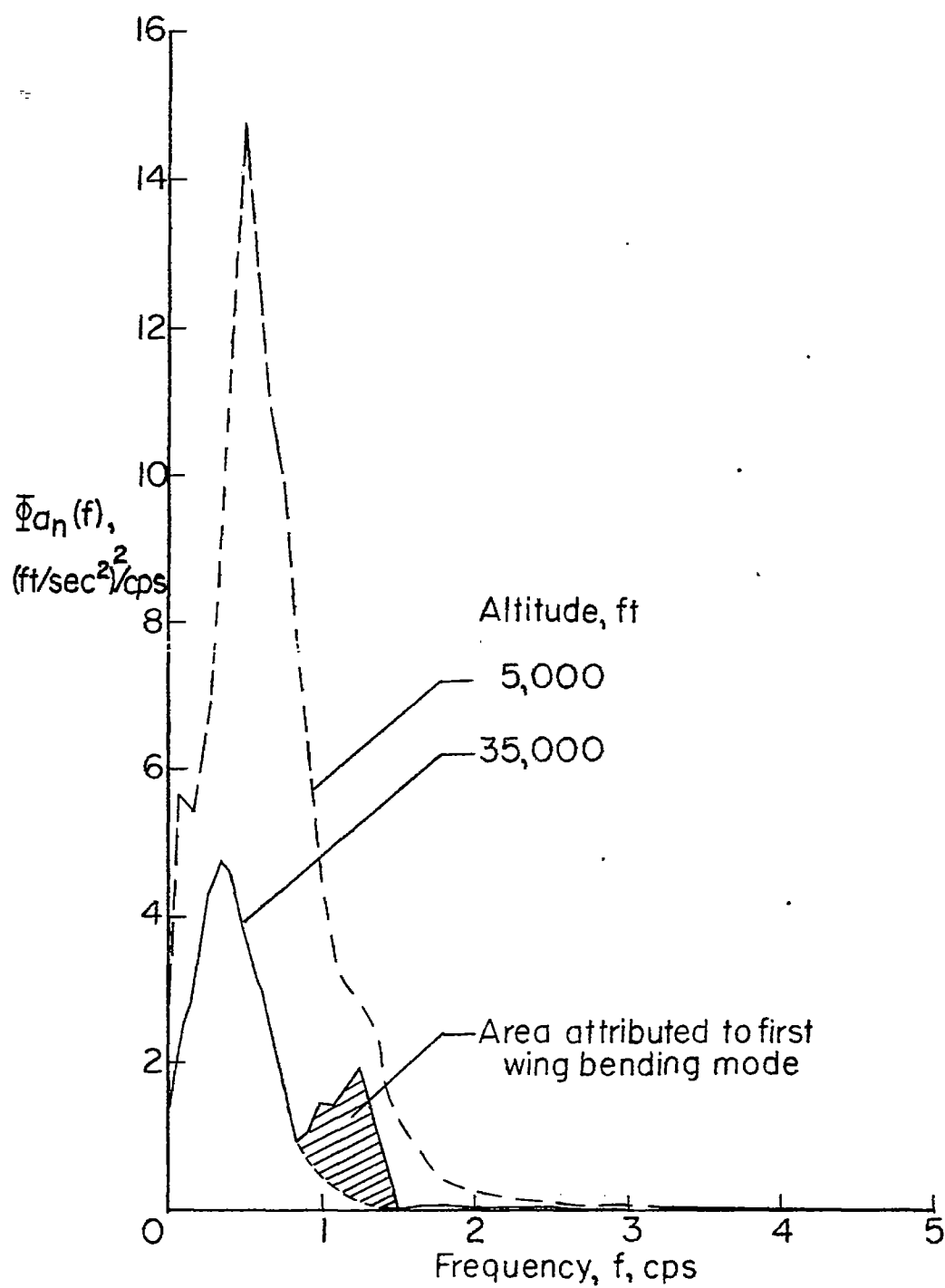


Figure 5.- Comparison of power spectrum of faired center-of-gravity acceleration for 5,000 feet and 35,000 feet altitude.

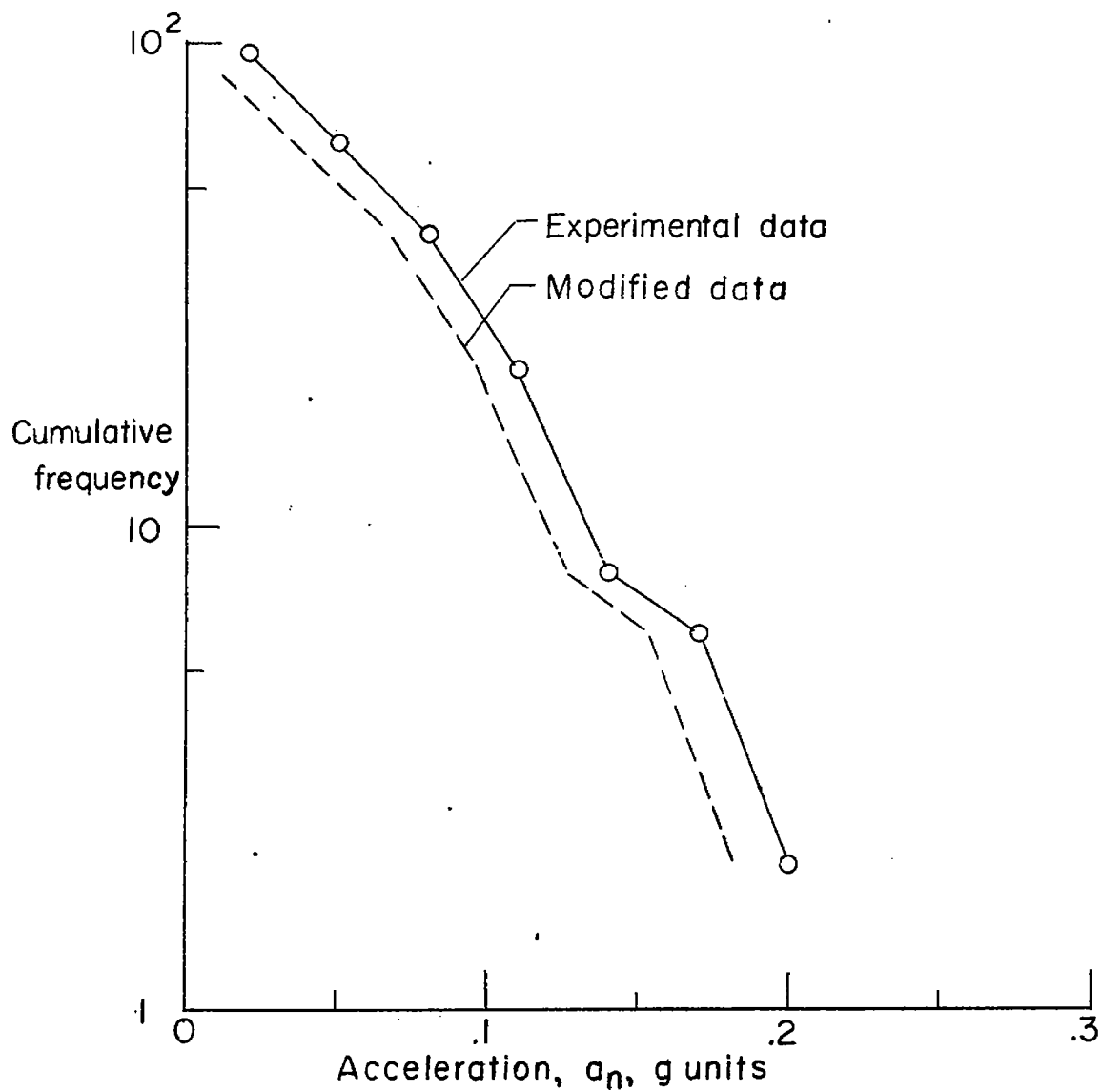


Figure 6.- Comparison of cumulative frequency distribution of faired center-of-gravity acceleration with the same distribution modified by use of equation 2.

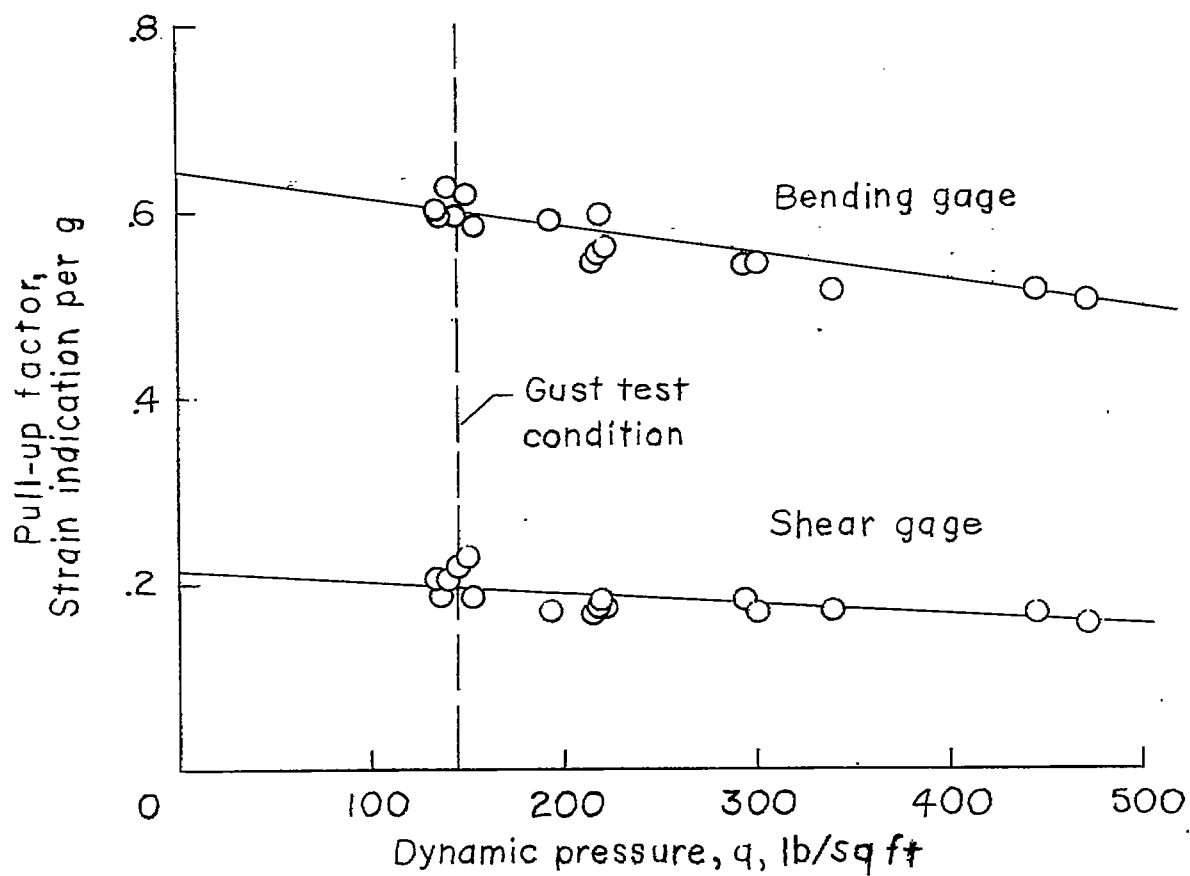


Figure 7.- Typical variation of pull-up factor with dynamic pressure.
Wing station 414; rear spar.

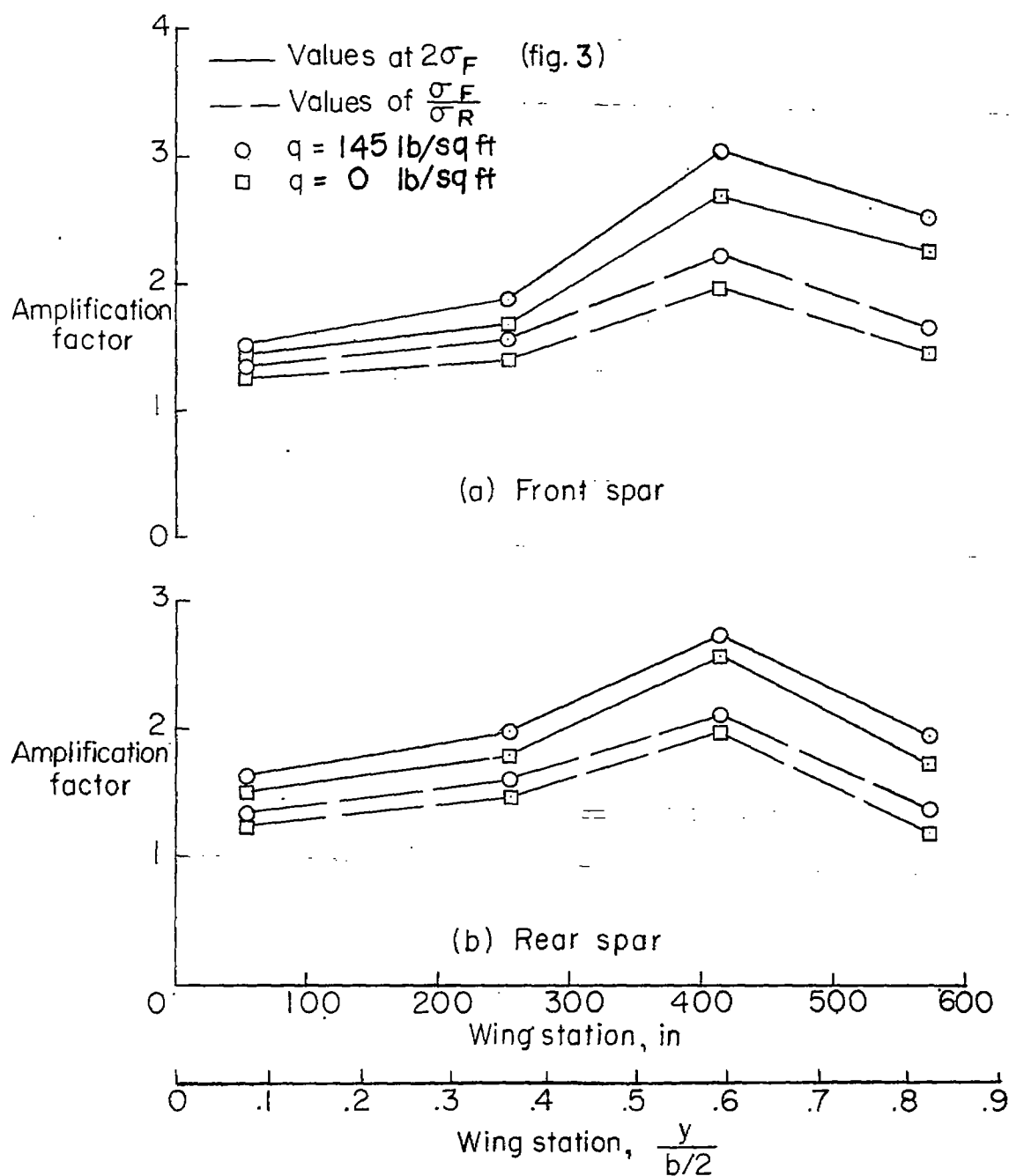


Figure 8.- Spanwise variation of amplification factor for bending strain.

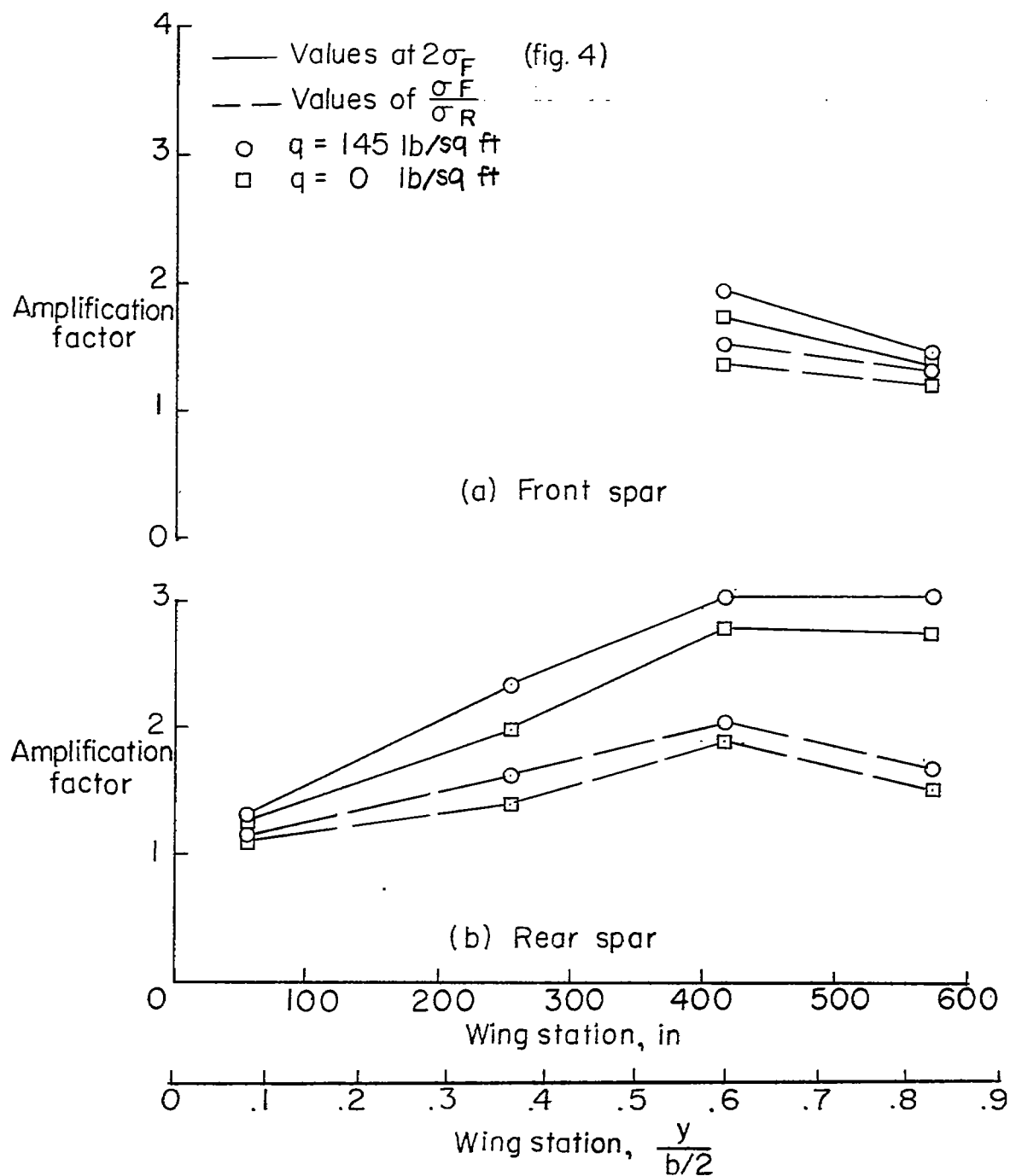


Figure 9.- Spanwise variation of amplification factor for shear strain.

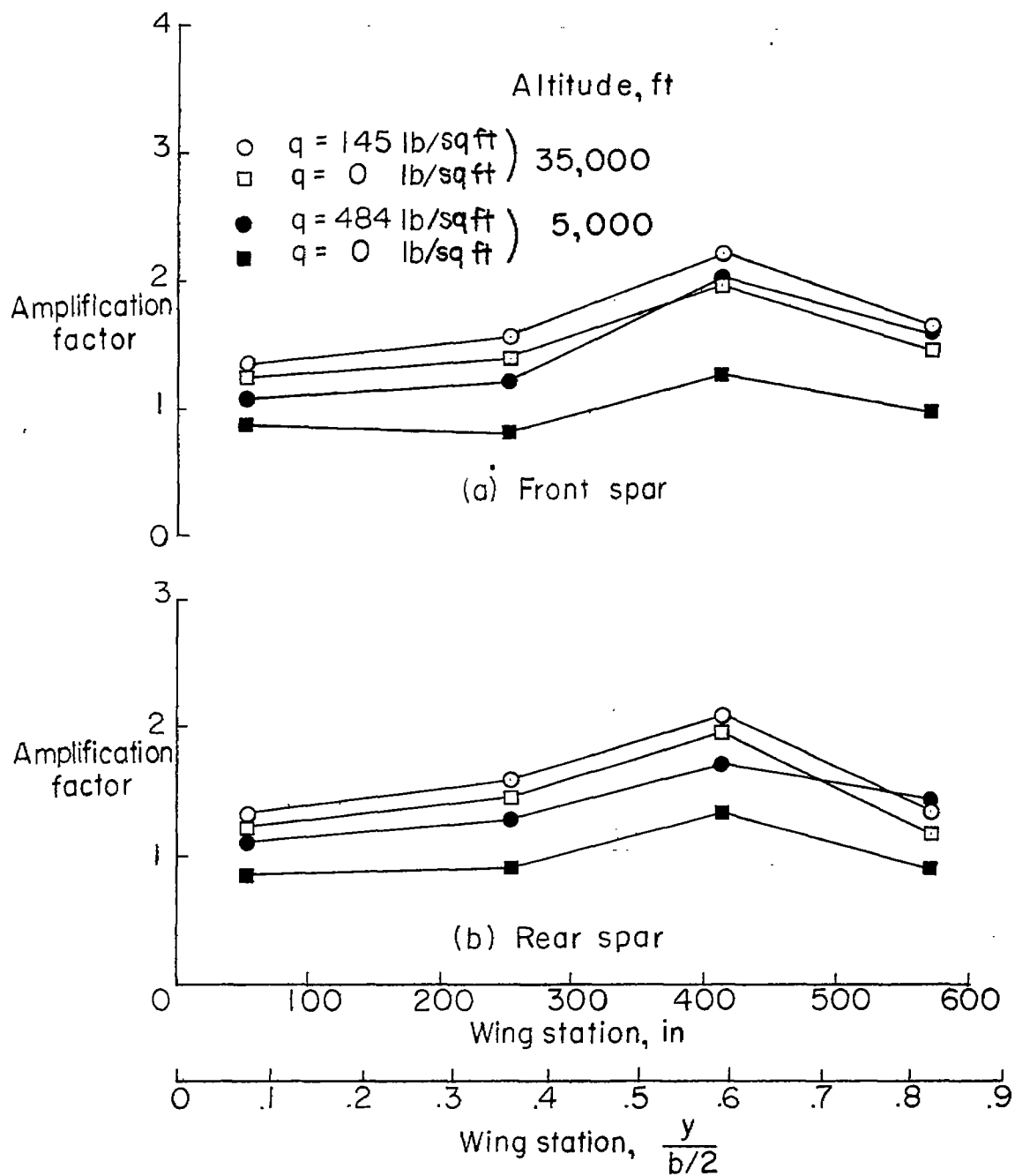


Figure 10.- Comparison of bending-strain amplification factors for two altitudes. Amplification factors determined from values of $\frac{\sigma_F}{\sigma_R}$.

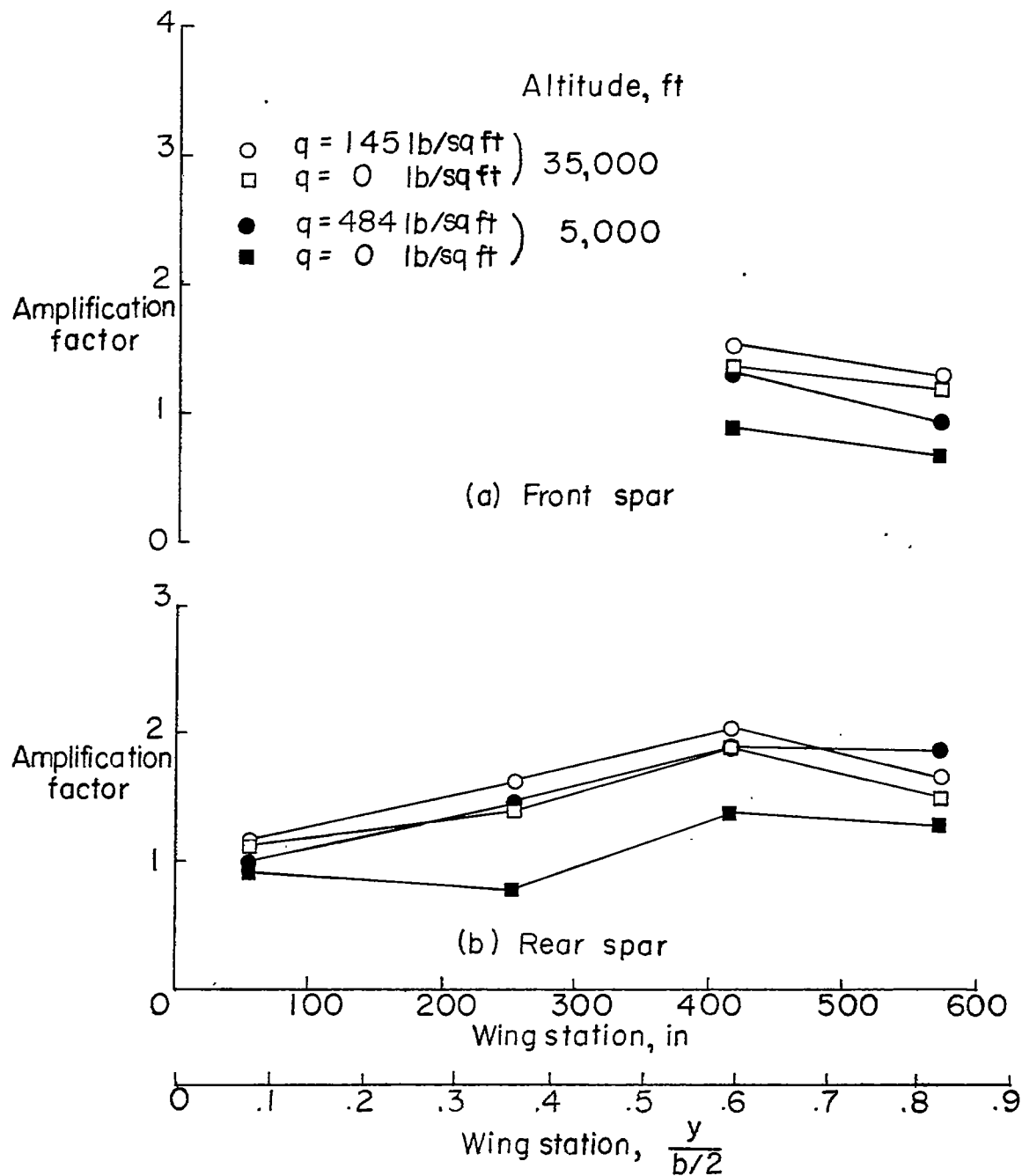


Figure 11.- Comparison of shear-strain amplification factors for two altitudes. Amplification factors determined from values of $\frac{\sigma_F}{\sigma_R}$.